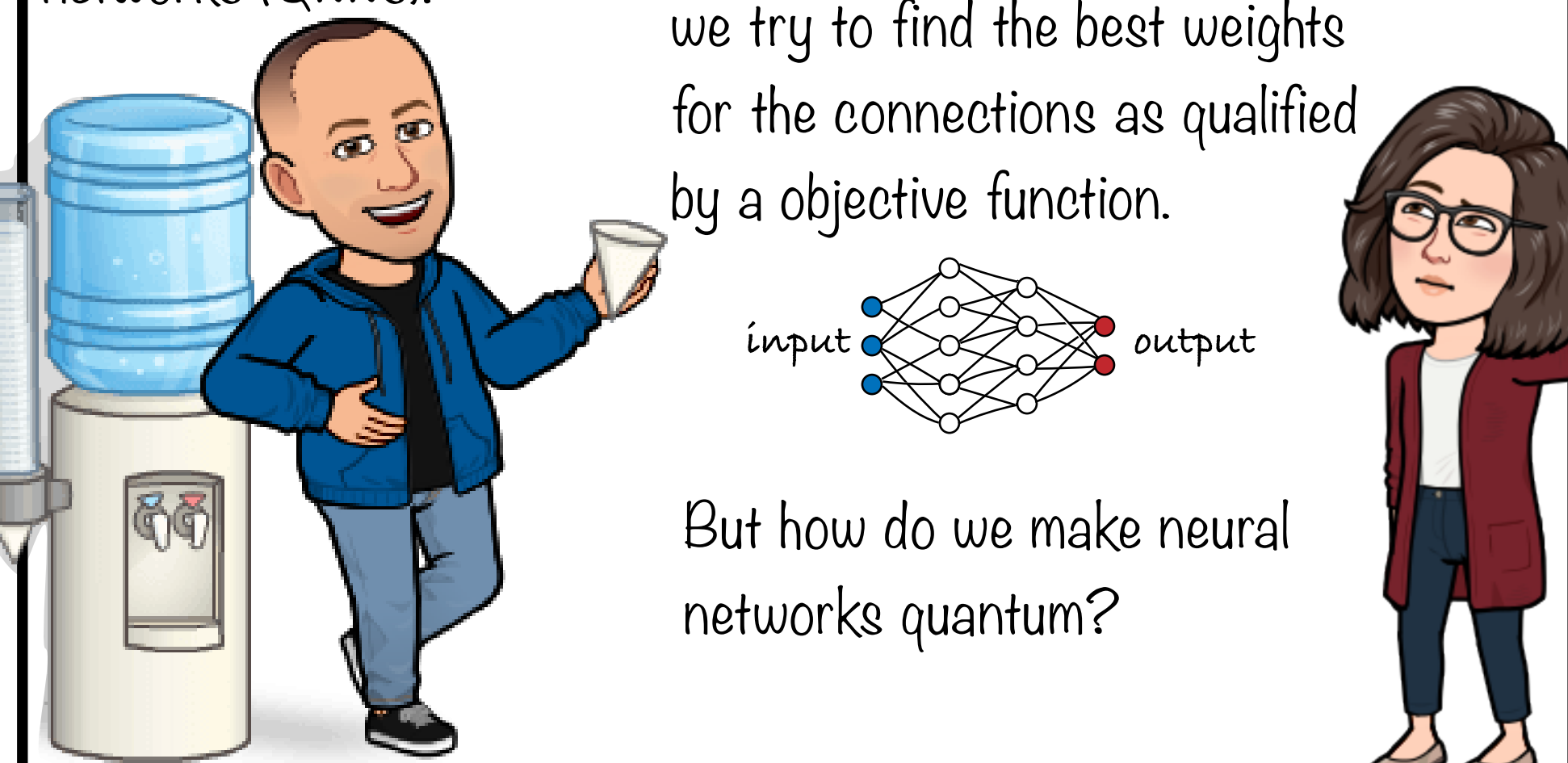


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# Entanglement Induced Barren Plateaus

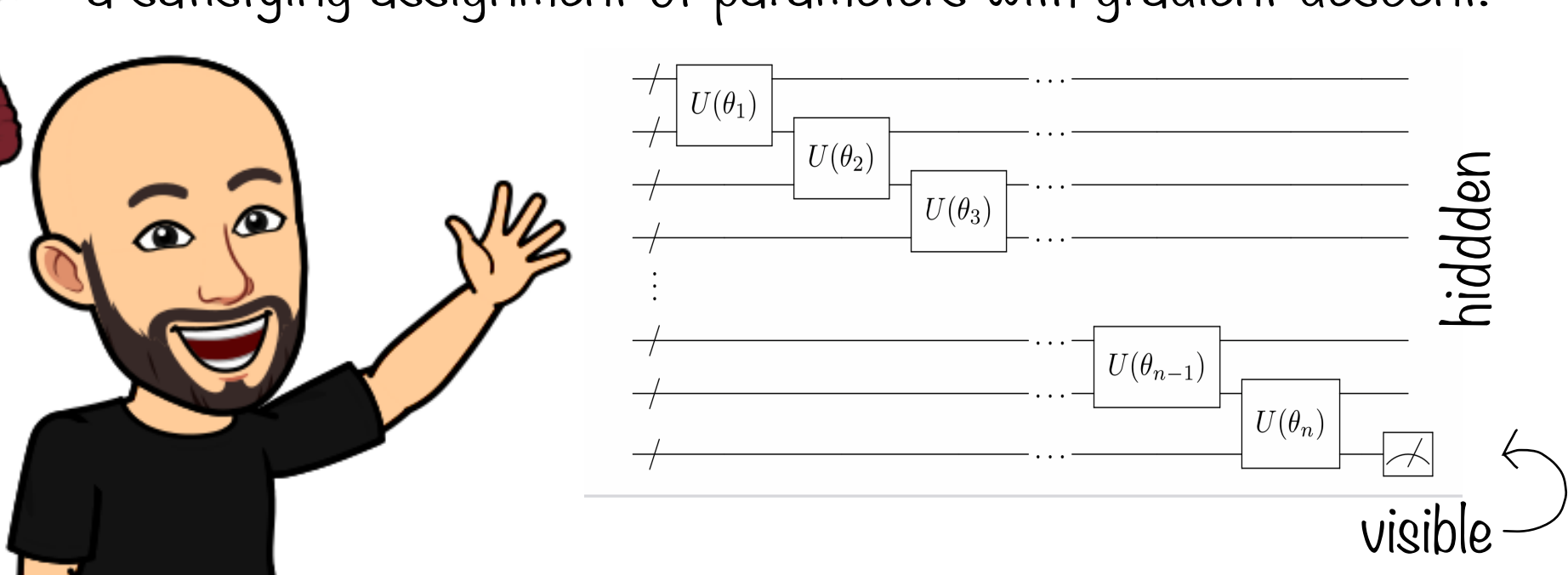
Let's train some quantum neural networks (QNNs).



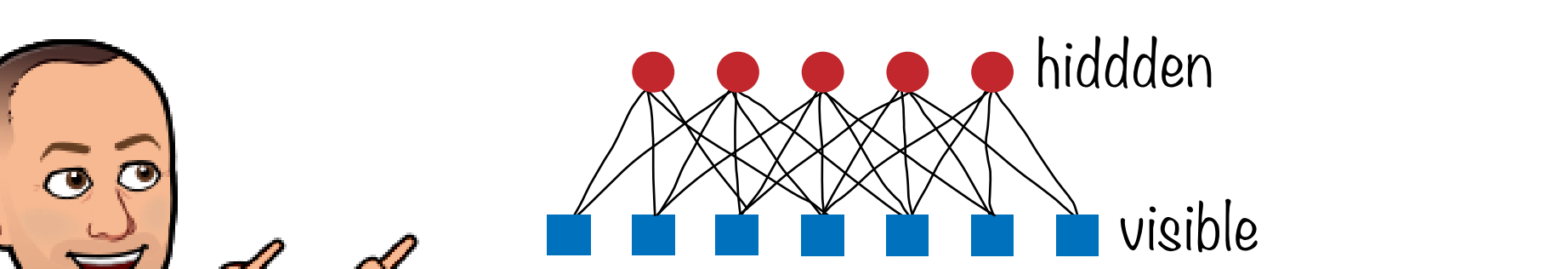
I know that traditional neural networks consist of layers of units. In the process of learning we try to find the best weights for the connections as qualified by a objective function.

But how do we make neural networks quantum?

There are several ideas. One realization of a QNN is a parametrized circuit. Each unit is represented by a qubit and at the end we measure only qubits corresponding to output units. We will call these qubits visible and the rest of the qubits hidden. We then try to find a satisfying assignment of parameters with gradient descent.



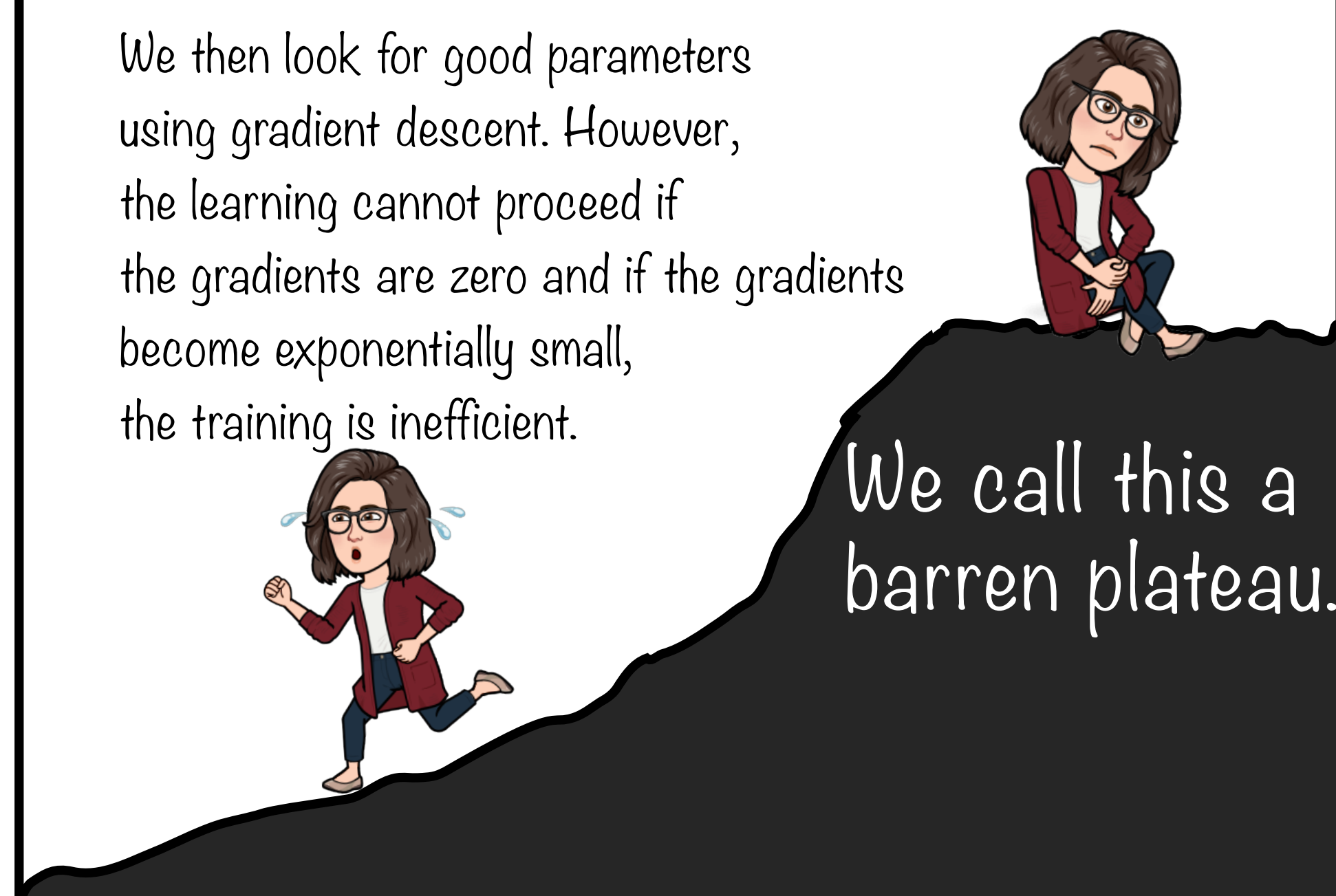
Another QNN is a quantum Boltzmann machine (QBM). QBMs are defined by a graph with an associated Hamiltonian.



The output state is  $\rho_v = \text{Tr}_h[\rho]$  where

$$\rho = \frac{e^{-H(\theta)}}{Z(\theta)} := \frac{e^{-\sum_i \theta_i H_i}}{\text{Tr}(e^{-\sum_i \theta_i H_i})}$$

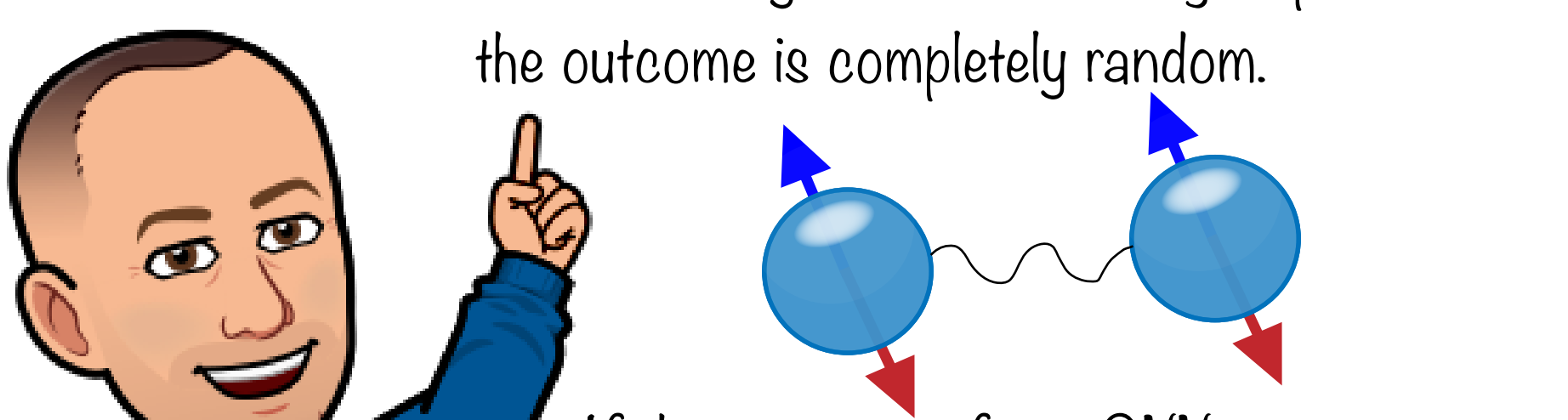
We then look for good parameters using gradient descent. However, the learning cannot proceed if the gradients are zero and if the gradients become exponentially small, the training is inefficient.



We call this a barren plateau.

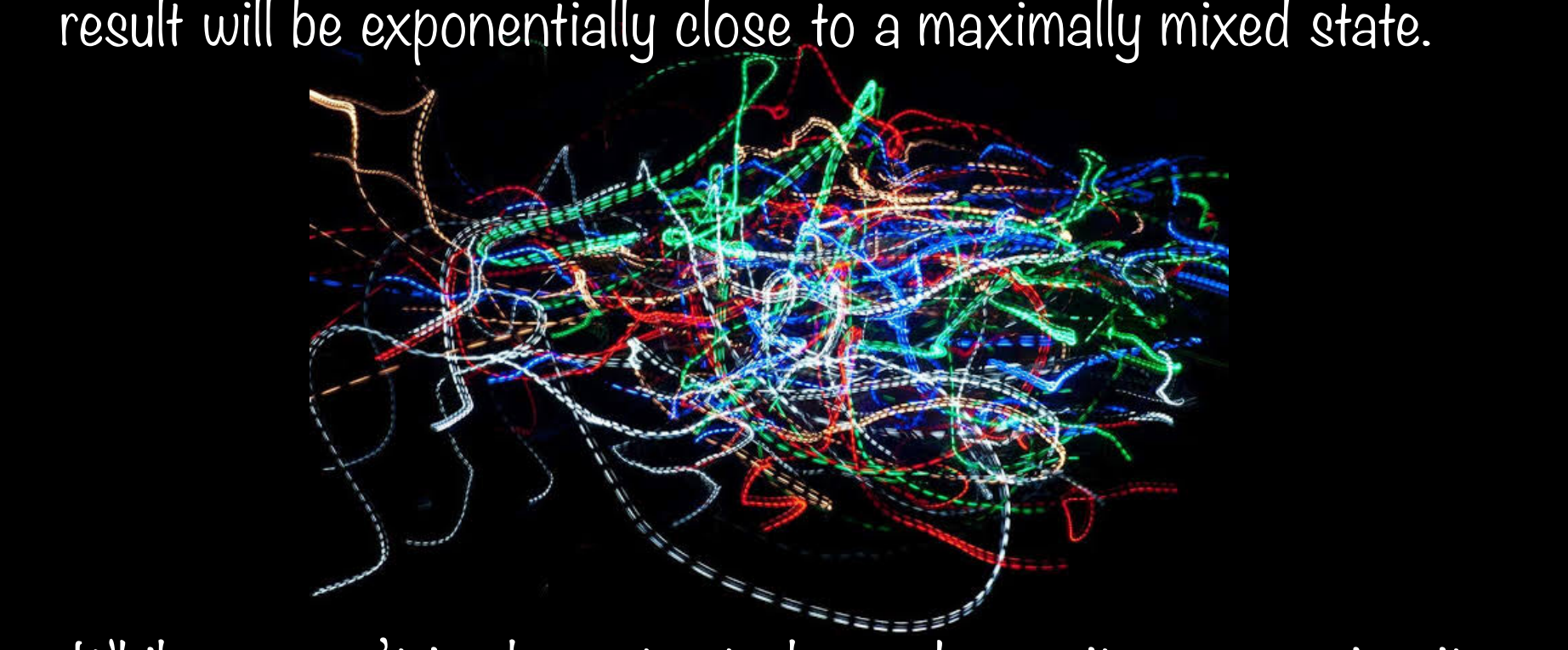
Barren plateaus are known to exist for various QNNs. Here we show that barren plateaus can be caused by excess entanglement between visible and hidden units.

If we measure only half of an entangled pair the outcome is completely random.



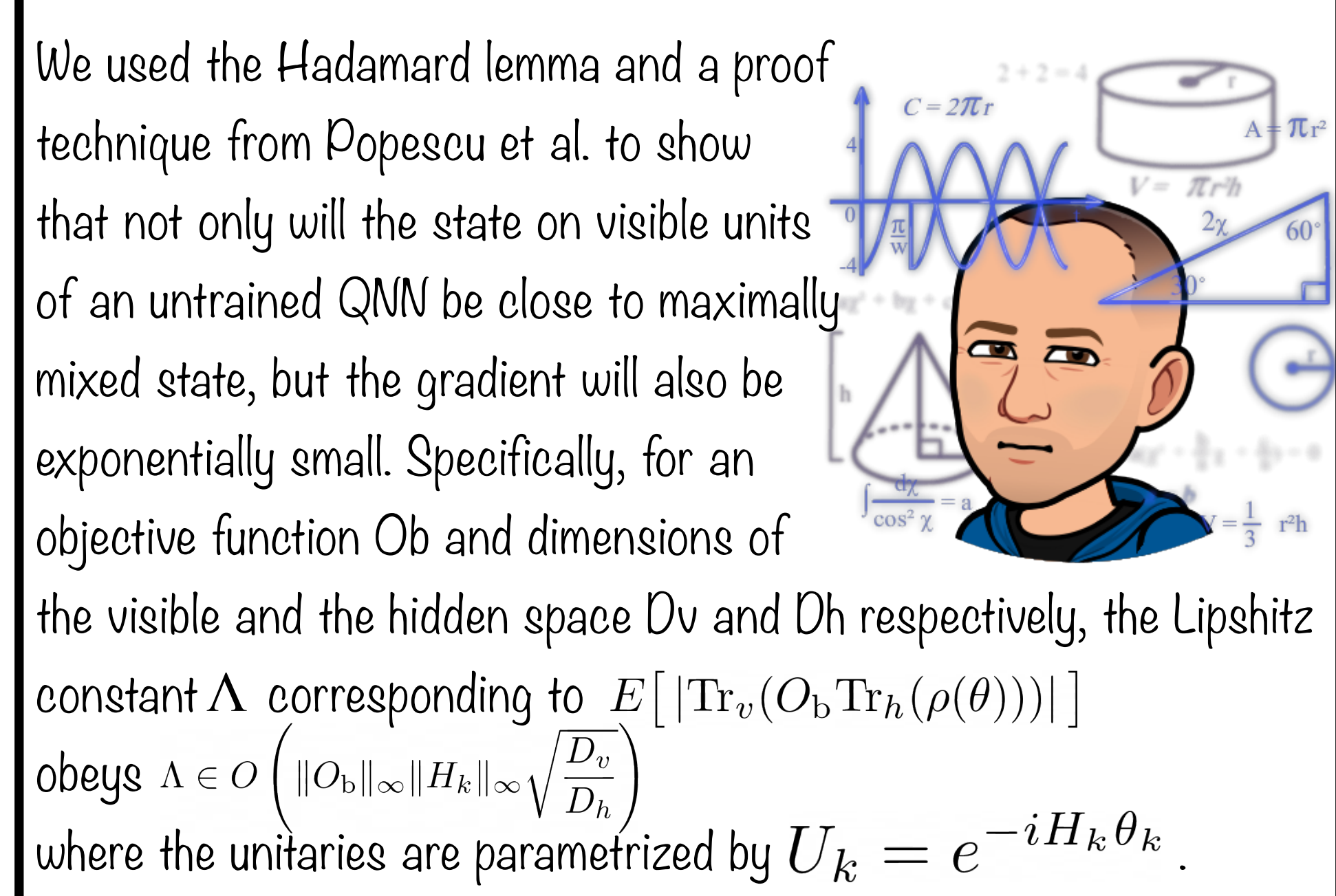
If the outcome of our QNN is very entangled, measuring only a small number of qubits will have the same effect.

Popescu et al. showed that if we apply a Haar-random unitary to a state and trace out a large subsystem, the result will be exponentially close to a maximally mixed state.

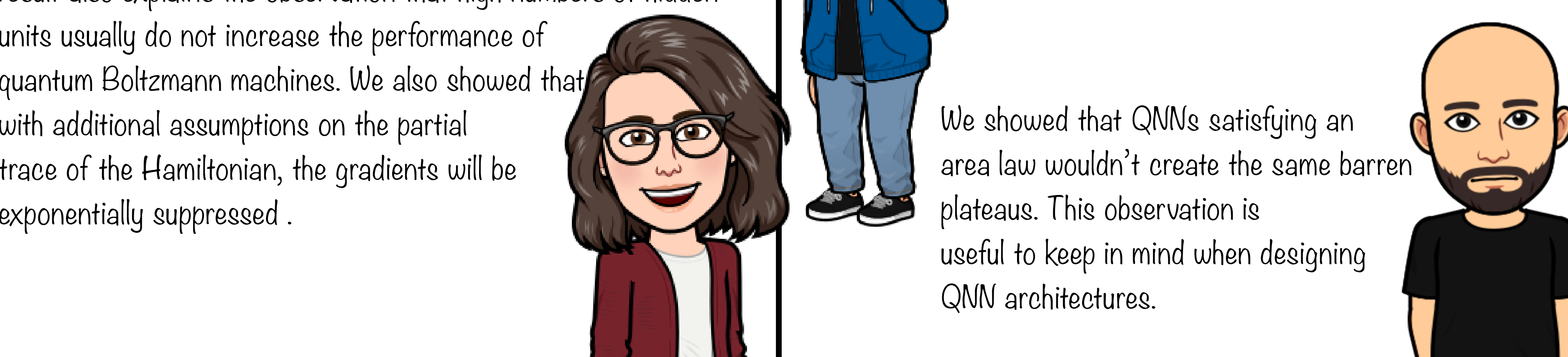


While we can't implement a truly random unitary as a circuit, untrained parametrized circuits can be well approximated by 2-designs.

We used the Hadamard lemma and a proof technique from Popescu et al. to show that not only will the state on visible units of an untrained QNN be close to maximally mixed state, but the gradient will also be exponentially small. Specifically, for an objective function  $O_b$  and dimensions of the visible and the hidden space  $D_v$  and  $D_h$  respectively, the Lipschitz constant  $\Lambda$  corresponding to  $E[|\text{Tr}_v(O_b \text{Tr}_h(\rho(\theta)))|]$  obeys  $\Lambda \in O\left(\frac{\|O_b\|_\infty \|H_k\|_\infty \sqrt{D_v}}{D_h}\right)$  where the unitaries are parametrized by  $U_k = e^{-iH_k \theta_k}$ .

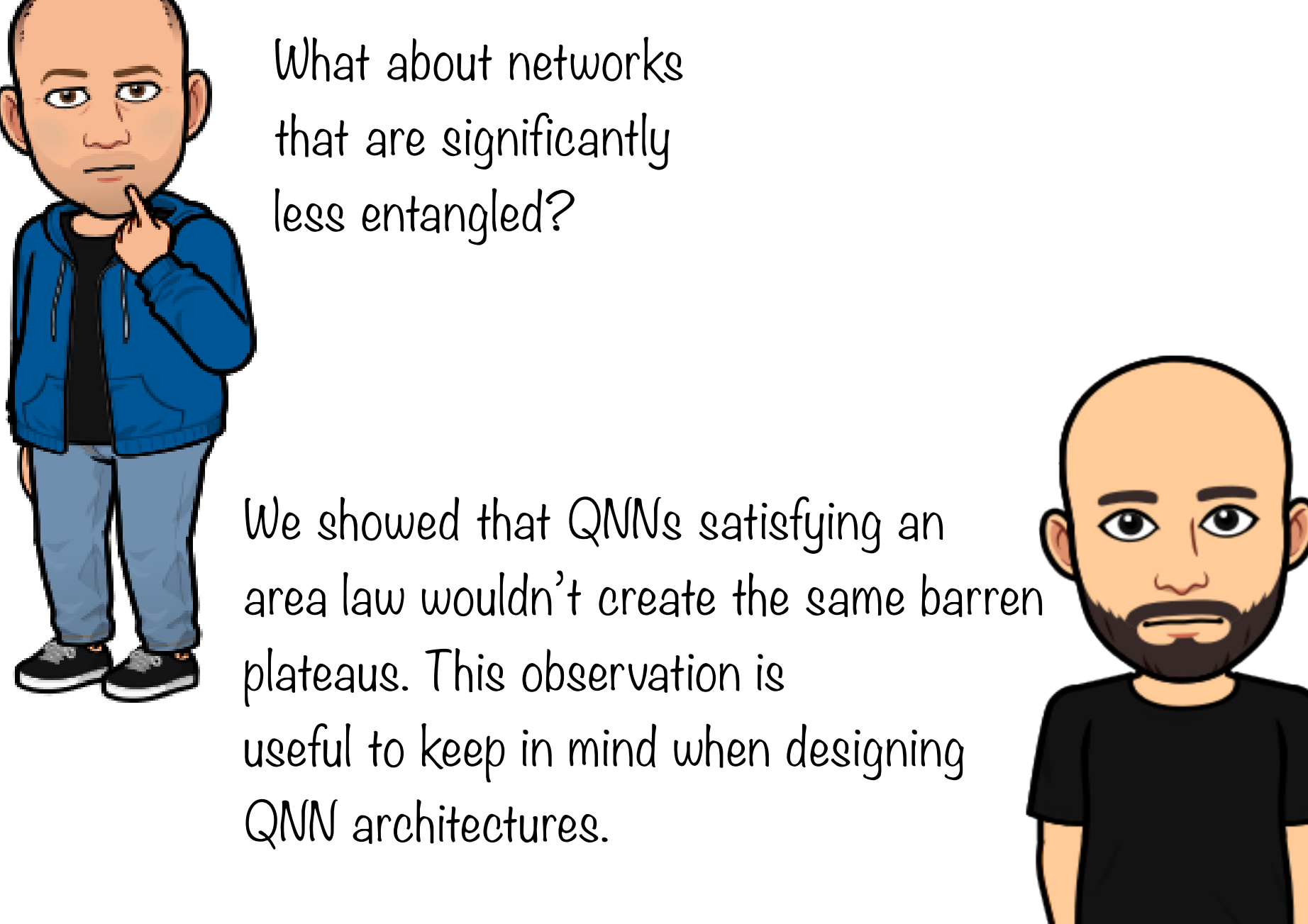


Using perturbation theory, we obtain a similar scaling for quantum Boltzmann machines. We showed that typical Hamiltonians generate thermal states that are close to the maximally mixed state. This result also explains the observation that high numbers of hidden units usually do not increase the performance of quantum Boltzmann machines. We also showed that with additional assumptions on the partial trace of the Hamiltonian, the gradients will be exponentially suppressed.

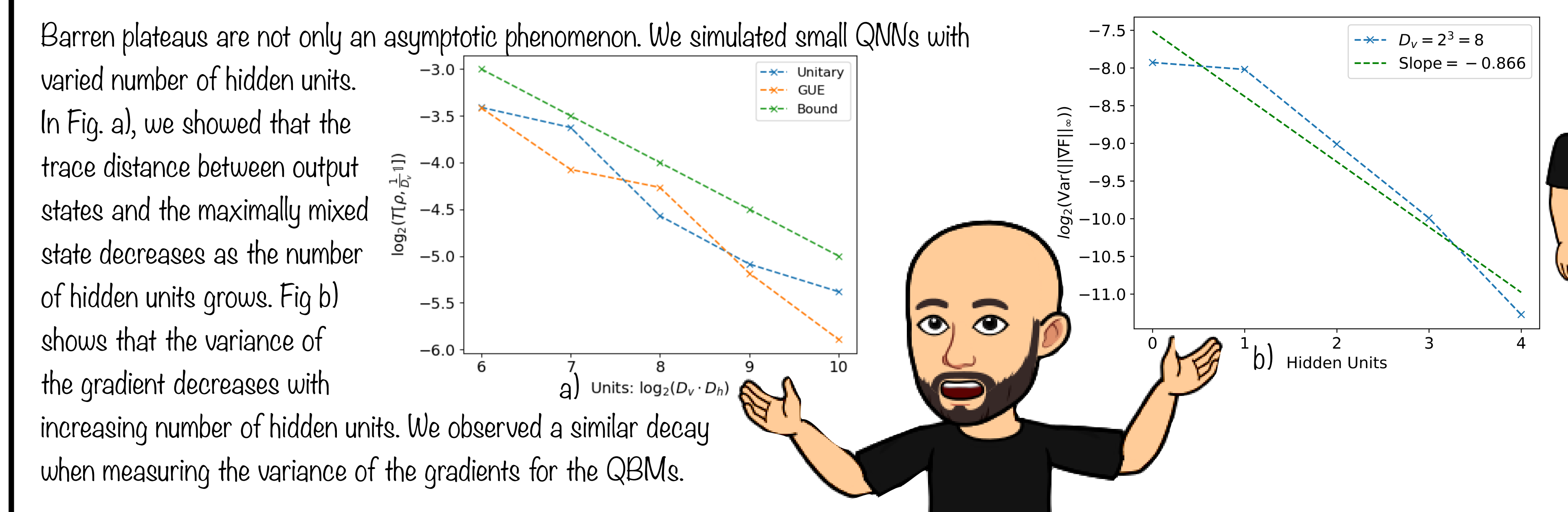


What about networks that are significantly less entangled?

We showed that QNNs satisfying an area law wouldn't create the same barren plateaus. This observation is useful to keep in mind when designing QNN architectures.

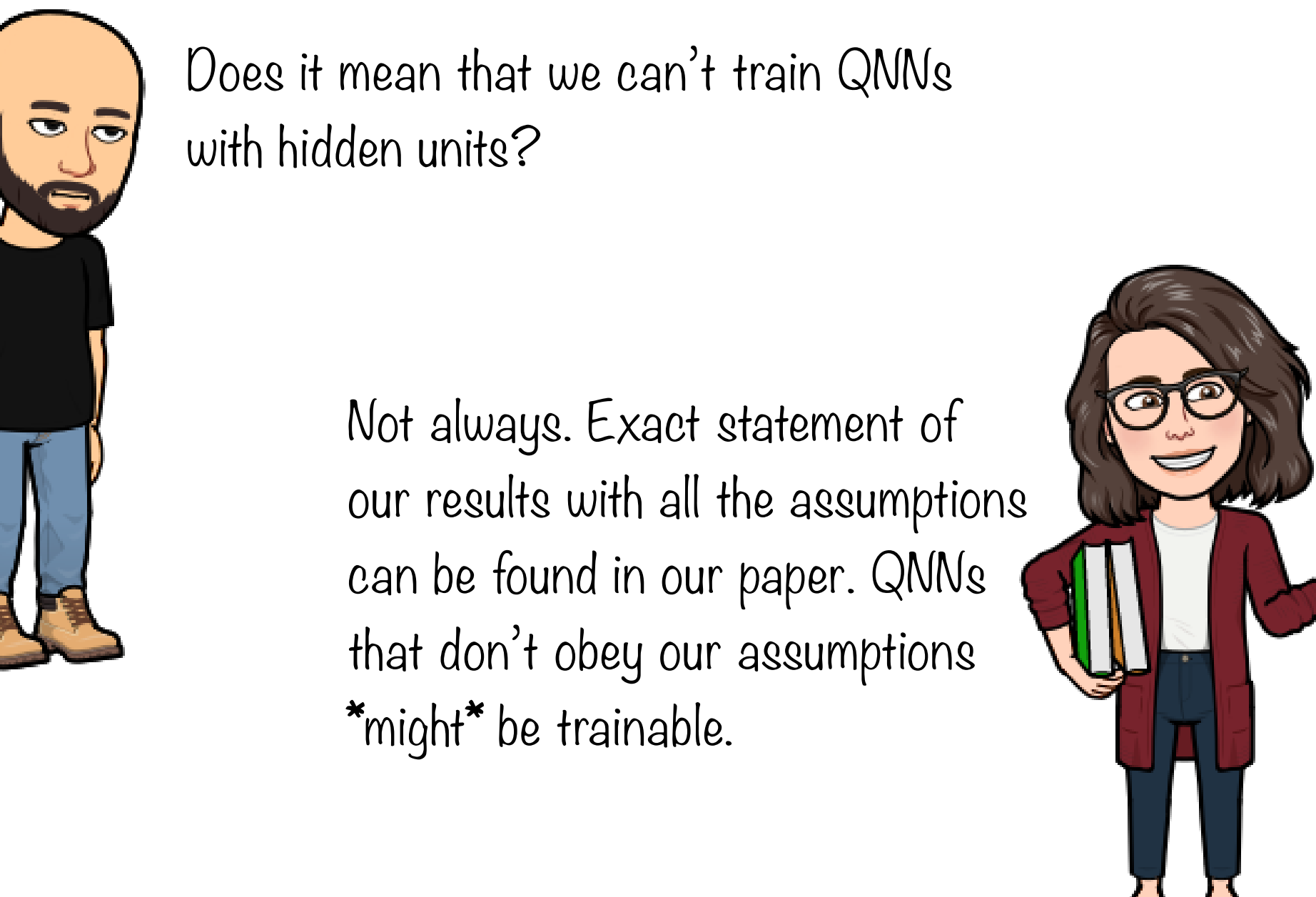


Barren plateaus are not only an asymptotic phenomenon. We simulated small QNNs with varied number of hidden units. In Fig. a), we showed that the trace distance between output states and the maximally mixed state decreases as the number of hidden units grows. Fig b) shows that the variance of the gradient decreases with increasing number of hidden units. We observed a similar decay when measuring the variance of the gradients for the QBMs.



Does it mean that we can't train QNNs with hidden units?

Not always. Exact statement of our results with all the assumptions can be found in our paper. QNNs that don't obey our assumptions \*might\* be trainable.



Read our paper <https://arxiv.org/pdf/2010.15968.pdf>

Or watch it as a talk on YouTube <https://youtu.be/8UIcFP0lEqc>



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