Methods in quantum computing

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August 8, 2023

University of Technology Sydney

Class overview

- UTS undergraduates + SQA grads (different background)
- 12 weeks, 3-hour interactive lecture per week
- 50% required to pass, attendance not required (but recommended)
- office hours immediately after class or by appointment

Information about the course

- Canvas
- most up to date:

  `www.mariakieferova.com/methods-in-quantum-computing`

Assessment

- 3 problem sets, each contributing 15% to the final grade
- group video project
- final presentation + report
- opportunities to earn bonus points through the term

Warning

- The content of this class is difficult (for an undergraduate class)
- We assume familiarity with quantum formalism (Introduction to quantum computing), Linear algebra as well as mathematical maturity

BUT:

- This is a small class, ask for help when needed!
- Take advantage of bonus problems.
- Gen in touch with me before the deadlines.

Help! I don’t remember quantum circuits anymore!

Optional "Background refresher, a.k.a Problem Set 0 (5/0):

- these problems should be quite easy for you - if they are not, they indicate what material you need to refresh
- due in/before the class in 2 weeks (August 22). Lecture #3
- solution will be released afterwards and any bonus points will be added to Problem Set 1

Problem set

- Individual assignment - submit your own work and explain your thinking
- Cite any used sources
- Programming (Python, Java, C, Matlab...) can help with some problems but it's not necessary. Submit your code if you're using it!

Other assignments

Group video:

- you'll be assigned partners and prepare a short video on one of the shortlisted topics
- a second part of the assignment is to assess 2 other videos

Final project:

- read at least a part of a scientific paper and present the gist of it at the last class
- prepare a report explaining the result

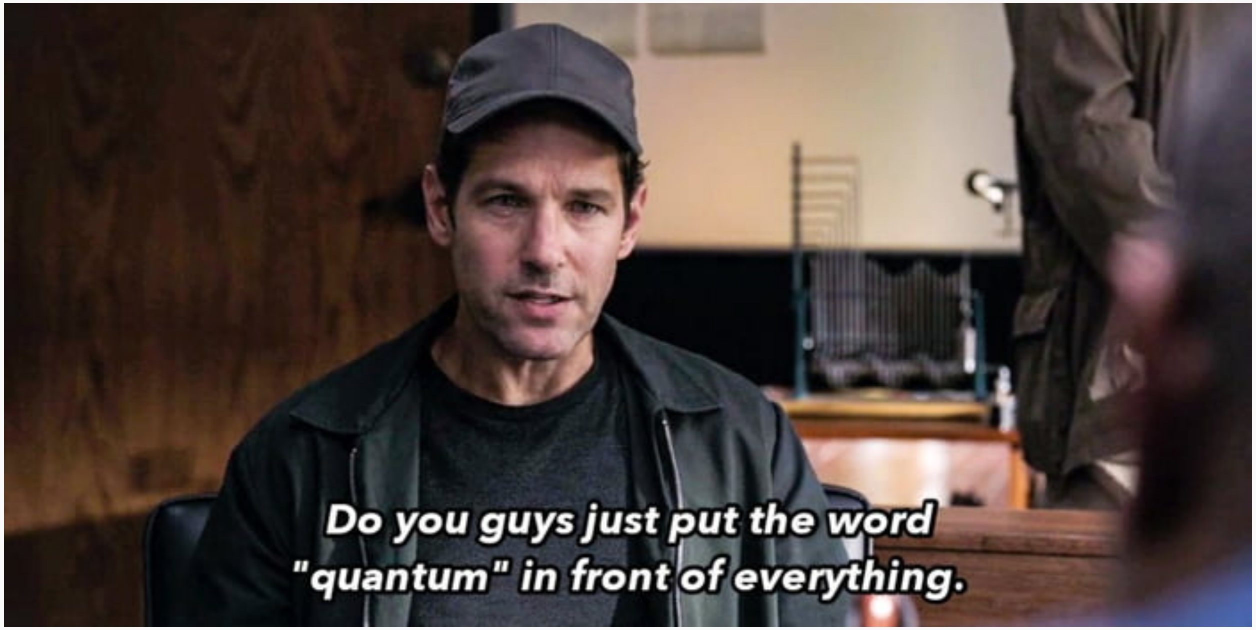
Code of conduct

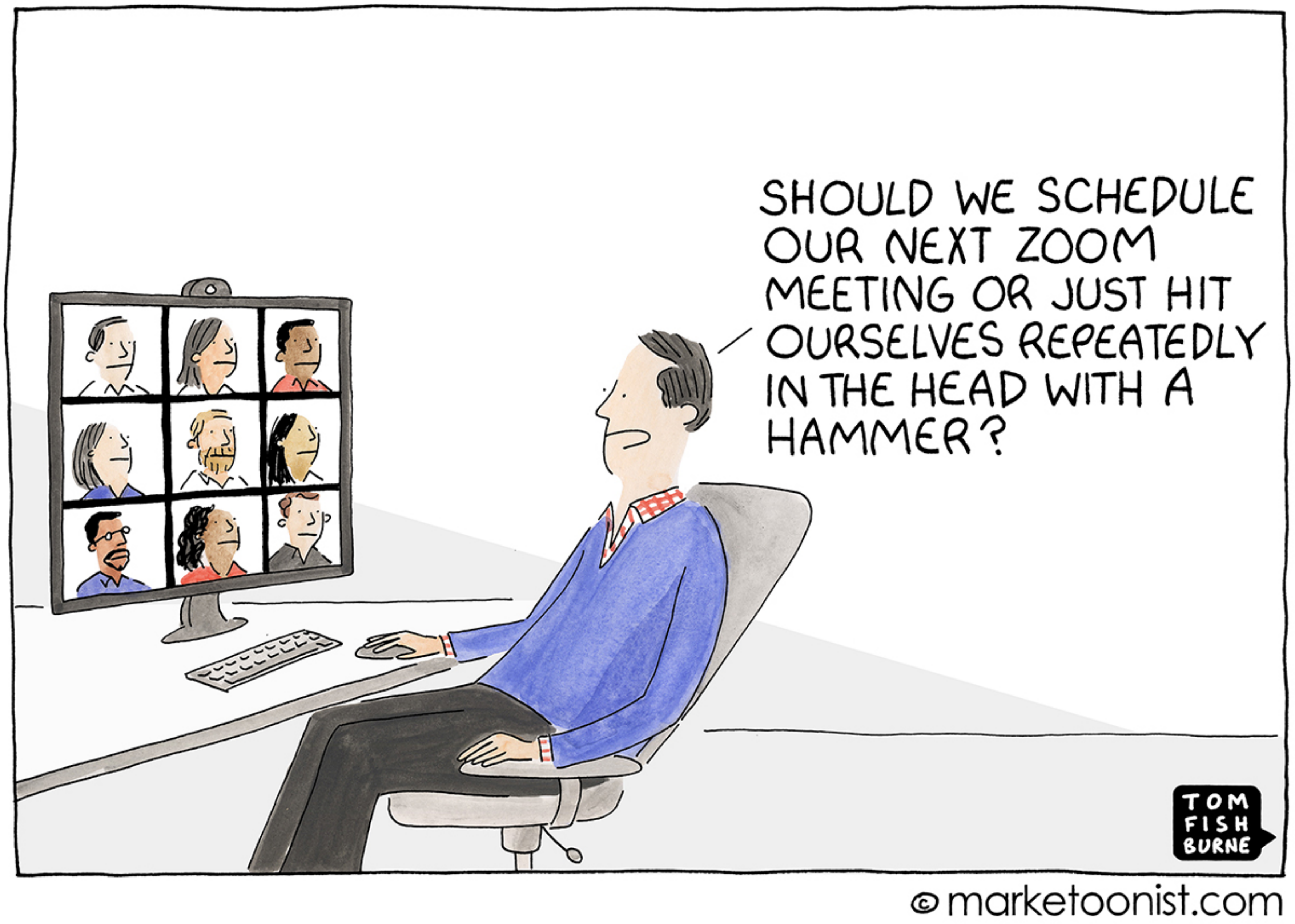
You (and I) are expected to follow UTS code of conduct - similar to CoC at other universities

- treat others with respect, create a safe learning environment
- no bullying or harassment
- no cheating, academic fraud or plagiarism

Topic overview

1. Quantum formalism, quantum mechanics and quantum information theory
2. Quantum stack - a bit of physics, architecture and quantum error correction
3. Quantum algorithm and complexity
4. Quantum communication and entanglement




SHOULD WE SCHEDULE OUR NEXT ZOOM MEETING OR JUST HIT OURSELVES REPEATEDLY IN THE HEAD WITH A HAMMER?
TOM FISH BURNE
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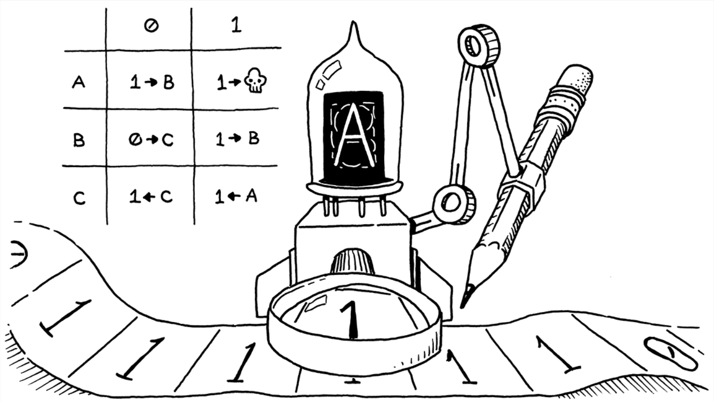
Introduce yourself to the class! Where are you from? What would you do if you were given 2 months off from your studies?

Today

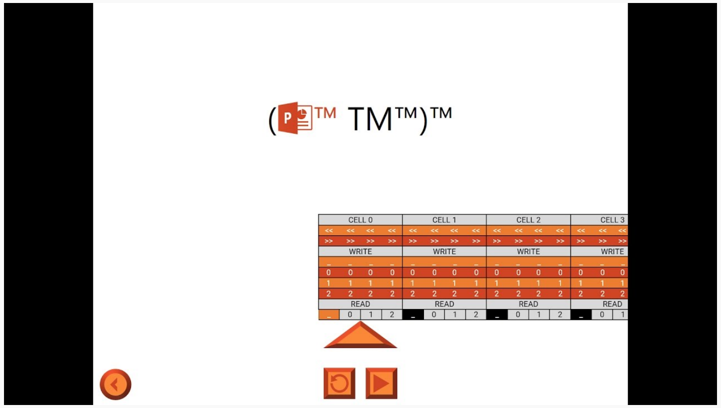
1. Motivation behind quantum computing
2. Models of computation
3. Quantum circuits

Computational models

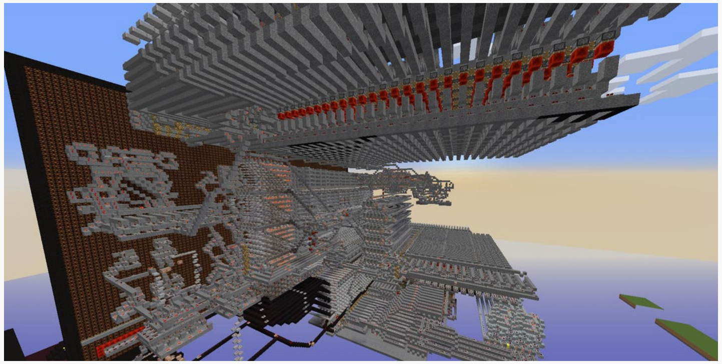


(a) Turing machine

(b) Conway's game of life

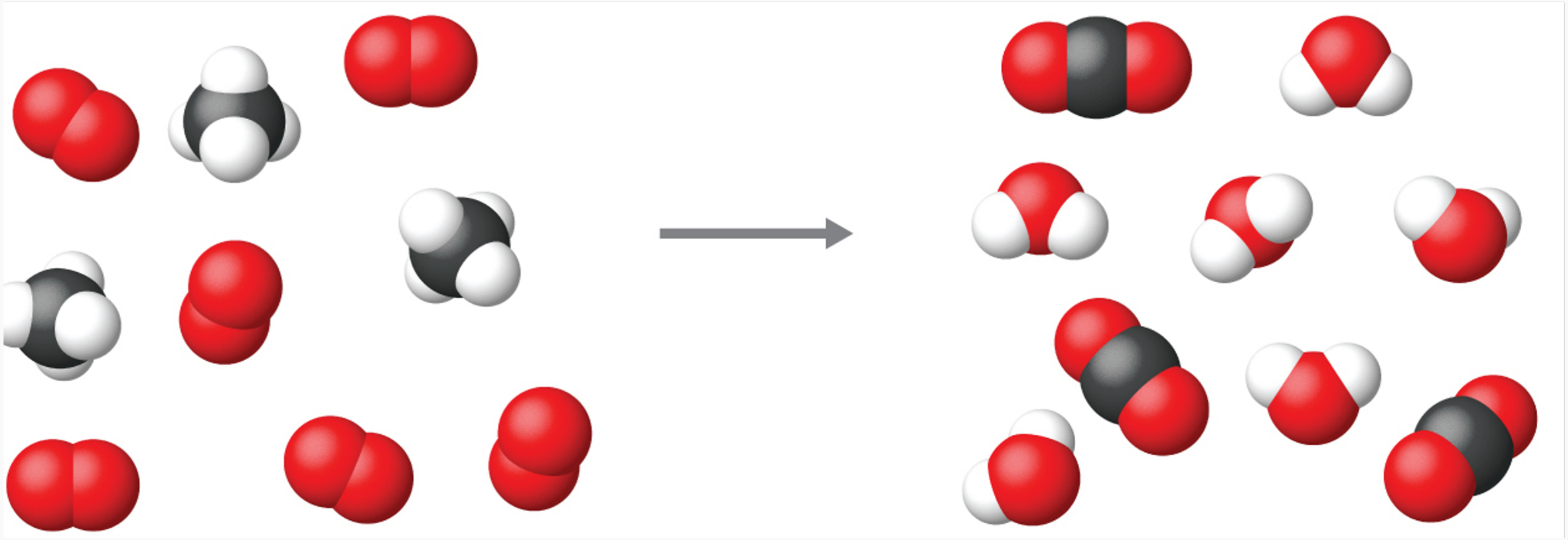


(c) Power Point



(d) Minecraft

Simulating nature

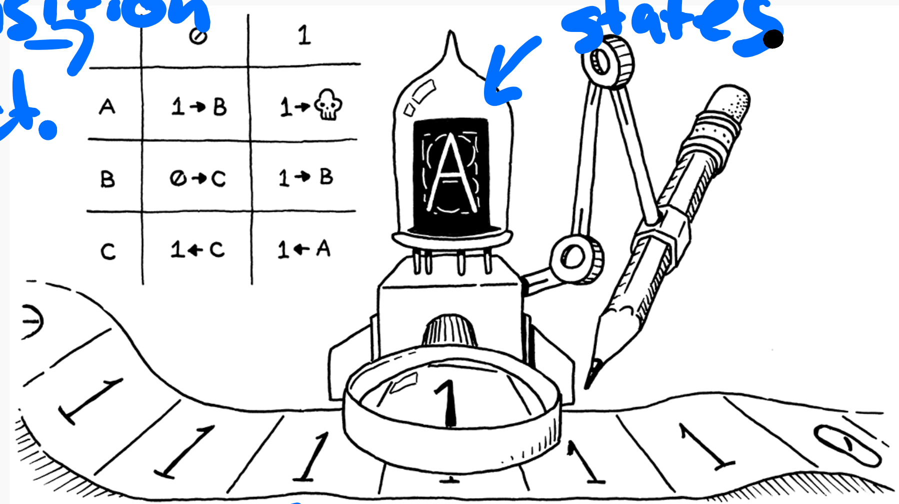


Physics of Computation

Why are you interested in quantum computing?

What topics are you working on and why did you choose them?

Turing machine

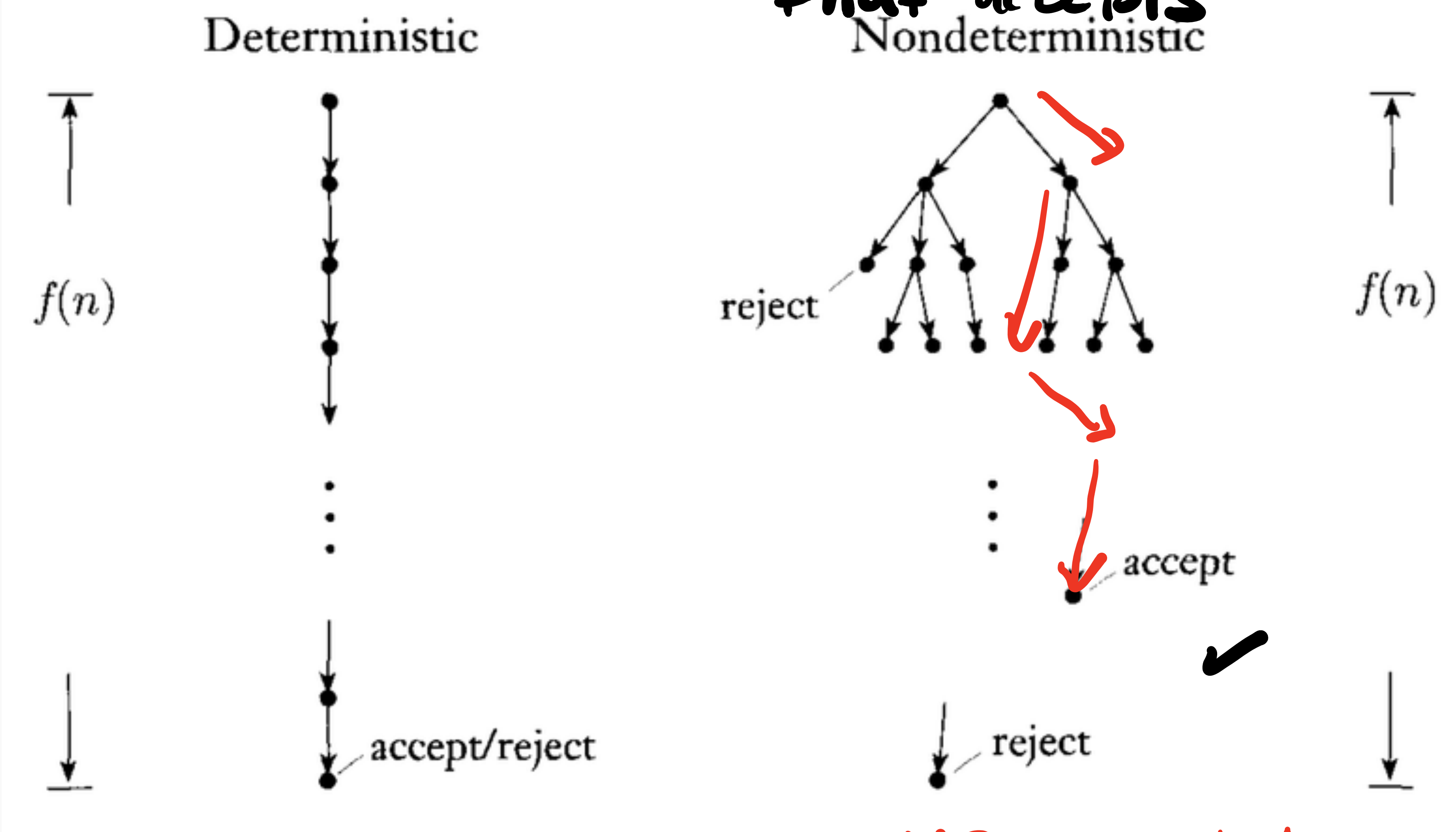


(0, 1, b)

- Γ a non-empty alphabet, i.e. a set of allowed symbols
- $b \in \Gamma$ a blank symbol
- $\Sigma \subseteq \Gamma$ a set of symbols that initially appear on the tape
- Q a finite set of states of the machine
- $q_0 \in Q$ the initial state
- $F \subseteq Q$ set of accepting states. If the TM reaches one of these states, the computation finishes and the input is accepted ("yes").
- $\delta : Q \setminus F \times \Gamma \to Q \times \Gamma \times \{\text{left, right}\}$ is the transition function.

Turing machines

→ there exist a calculation that accepts



P - polynomial

NP nondetermin. polynomial

Church-Turing thesis

no restrictions on time or space (tape of the TM)

A Turing machine can simulate any realistic model of computation.

include quantum computers

Extended Church–Turing thesis

If quantum computers can be built, then the extended CH-T thesis is not true.

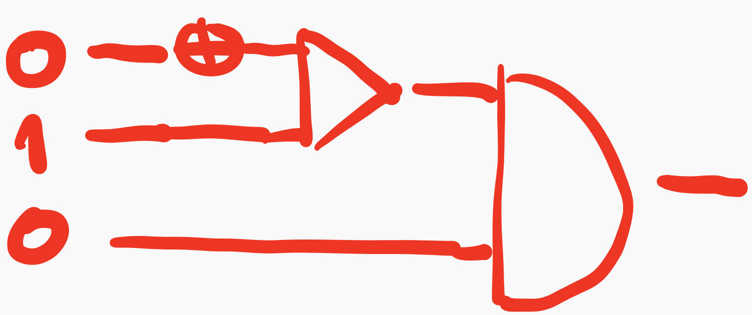
with poly overhead

A probabilistic Turing machine can efficiently simulate any realistic model of computation.

can toss a biased coin before every step

Logical circuits

Denote $\mathbb{B}^n := \mathbb{Z}_2^n$. Let $f : \mathbb{B}^n \to \mathbb{B}^m$ be a Boolean function that takes an n-bit string as input and outputs an m-bit string. Let $\mathcal{G}$ be a collection of basic logic gates. A Boolean circuit for f is a sequence of gates $\{g_1, \cdots, g_L\} \in \mathcal{G}$ which converts an input $\boldsymbol{x} \in \mathbb{B}^n$ to the output $\boldsymbol{y} \in \mathbb{B}^m$ with a fixed size of K auxiliary bits.



Logical gates

table in the lecture notes

NAND = AND + NOT

Uniform circuit families

function (a+b)

circuit for 2 bits
3 bits
4
5
⋮
10
100
⋮
n bits

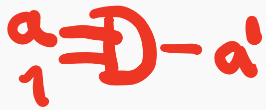
recipe to prepare circuits for different sizes of the input

Exercise

meet at 5:05

Show that the NAND and FANOUT (copy) are universal for computation, i.e. they can be used to express all possible truth tables.

(a) Show that the NOT gate can be simulated using a single NAND gate.



(b) Show that the AND gate can be simulated with a constant number of NAND gates.

(c) Show that the OR gate can be simulated with a constant number of NAND gates. Hint: in the footnote [1]. How many NANDs is required for this construction?

<u>You might use additional bits</u> initialized to 0 or 1.

[1] Use De Morgan's Law: A OR B = NOT (NOT A AND NOT B).

Reversible circuits

Logical circuits that can be inverted are known as reversible circuits.

a → f(a)

given(a ∧ b) what was a?

We can compute a from f(a)

Exercise

What operations in Table 1 are reversible? What are the inverse operations to the reversible gates in Tables 1 and 2?

Qubits 1 qubit

| ⟩ ket   ⟨ | ⟩ braket – inner product
⟨ | bra   – number

Dirac notation

$$0_L \to |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$1_L \to |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$\langle 0| = (1 \;\; 0)$
$\langle 1| = (0 \;\; 1)$

Hello

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

superposition – linear combination

where $|\alpha|^2 + |\beta|^2 = 1$ normalization

L^2

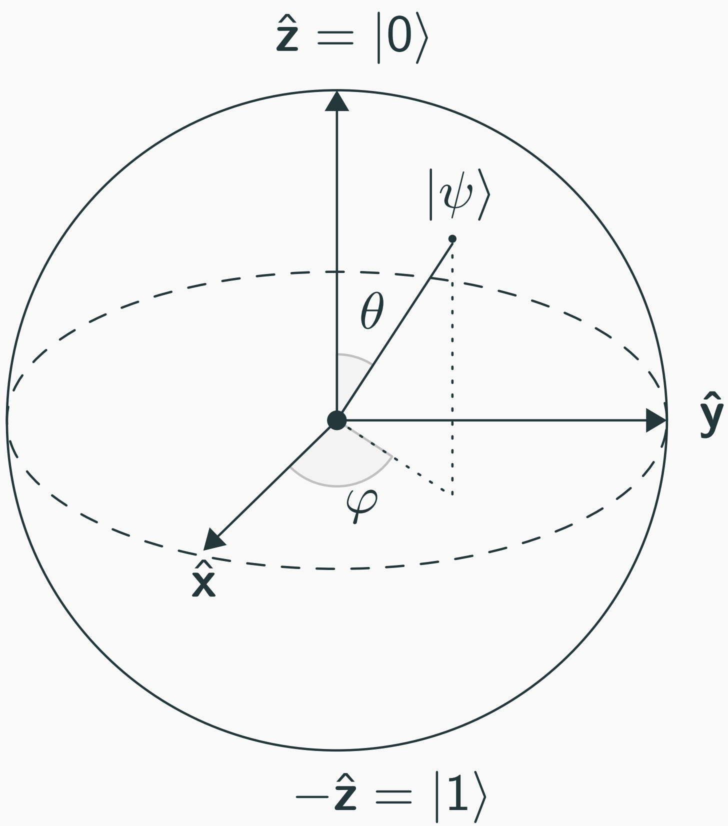
Qubits

pure state → poin on a sphere
Bloch

any pure state on 1 qubit can be defined by φ, θ

$$|\psi\rangle = \cos\frac{\theta}{2}\,|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}\,|1\rangle\,.$$

real number



|0⟩ - pure state

Quantum operations

an operation

Given an initial state $|\psi_0\rangle$, we can apply a gate U to obtain a new state $|\psi_1\rangle$

$$|\psi_1\rangle = U\,|\psi_0\rangle\,.$$

$$|\psi_2\rangle = U_2|\psi_1\rangle = U_2 U_1|\psi\rangle$$

Exercise

1 qubit is enough

Show that quantum operations must be unitary in order to preserve the norm of quantum states. An operation U is unitary if and only it satisfies $U^{-1} = U^{\dagger}$.

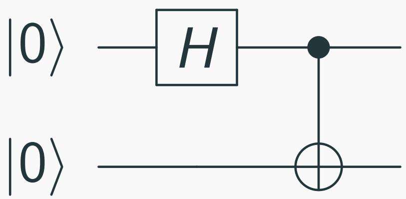
Exercise

a Show that for a unitary matrix U, $|\det(U)| = 1$. Hint: in a footnote.

b A global phase of a quantum state is not detectable. In other words, states $|\psi\rangle$ and $e^{i\psi}|\psi\rangle$ represent the same physical state. What consequence will it have for single qubit gates?

c Write the most general single qubit gate U using the convention $\det(U) = 1$.

What is the state state prepared by this circuit?



$$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

$$|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1$$

$$|00\rangle = |0\rangle|0\rangle = |0\rangle\otimes|0\rangle$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle-|1\rangle) = |-\rangle$$

$$|0\rangle|0\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \xrightarrow{CNOT} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

NOT

Given some unitary U, how would we implement it?

H + Toff + ancillas initialized to 0 or 1

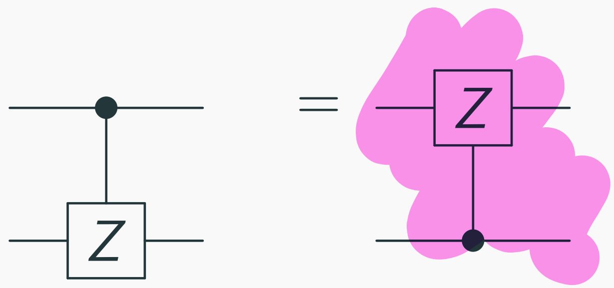
H, CNOT, T

Given any universal set of gates $\mathcal{G}$ that is closed under inverse, any unitary operation $U \in SU(d)$ can be ε-approximated using $O(\log^c(\frac{1}{\varepsilon}))$ gates from $\mathcal{G}$ for some constant c.

Scaling in error only

Exercise

1. Show that



2. Show that $HZH = X$ and $HXH = Z$.

a) show that the circuits output the same for all basis states

b) write gates as matrices, multiply them

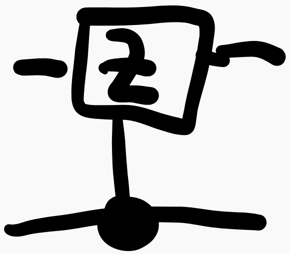
c) $U|\psi\rangle = |\phi\rangle$ $\quad U = |\phi\rangle\langle\psi| + |\phi\rangle\langle\psi|^{\dagger}$

$$|0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes Z = CZ$$

$$|0\rangle\langle 0| \otimes (|0\rangle\langle 0| + |1\rangle\langle 1|) + |1\rangle\langle 1| \otimes (|0\rangle\langle 0| - |1\rangle\langle 1|) =$$

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| \quad \text{LHS}$$



$$= \mathbb{1} \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$$

$$= (|0\rangle\langle 0| + |1\rangle\langle 1|)|0\rangle\langle 0| + (|0\rangle\langle 0| - |1\rangle\langle 1|) \otimes |1\rangle\langle 1|$$

$$= |00\rangle\langle 00| + |10\rangle\langle 10| + |01\rangle\langle 01| - |11\rangle\langle 11| \quad \text{RHS}$$

LHS = RHS

THE END